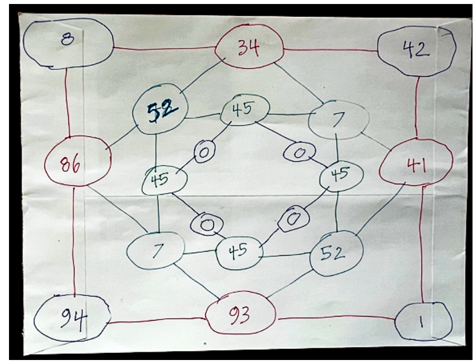
Diffies

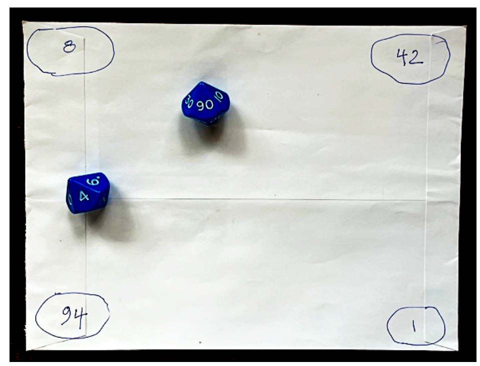
Diffies are a natural extension of race games. Like the race back down to zero in the second part of a race game, diffies end when all the differences between the corners equal zero.[1] Like race games, diffies exercise – and + concepts and skills, composition and decomposition of base ten units,[2] but students typically do diffies by writing on paper rather than by manipulating base ten blocks on place value mats.[3] (Nevertheless, concrete manipulatives can be used to support or verify paper-and-pencil calculations.) Diffies reveal the tight relationship between subtraction and addition, the notion of *additive differences*—exactly what students work with while playing race games, especially in the 2nd half of any race game: the race down to zero. To find the difference between two numbers, i.e., the *gap* between the numbers on the number line, you can add up from the smaller one or subtract down from the larger one. Either way, your result will be the same. Put algebraically, $A+B = C \Leftrightarrow C-B = A$ and $C-A = B$ for $A, B, C > 0$.



A finished diffy

When subtracting, just as in a race game, using the standard algorithm for subtraction or the base-ten-blocks equivalent, students may need to borrow, trade, or regroup, and then check their work with addition.

And when they're doing additive differences, students will need to make larger denominations and keep track (on paper or in their heads) of what it took to get from the lesser number to the greater.[4]



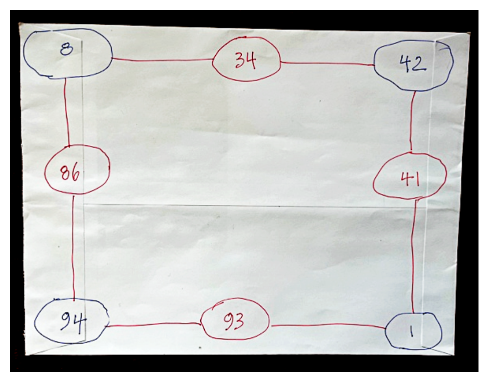
First step: Put numbers in the corners

How to do a diffy

1. Get a sheet of paper and put numbers in each corner.

I use ten-sided dice ones (0-9) and tens (00-90) to make sure that those numbers are random. (Kids like to roll the dice too—you get agency and affect in one convenient, random package!) For a 3-digit diffy, I might use a hundreds die, but I often customize a blank 6-sided die with 0, 1, or 2 on the faces to keep the corner numbers less than 300.

2. Find the differences between the corner numbers and put those differences in between the corner numbers, connected with short lines. (Find differences either by taking the smaller number from the larger or by adding up to the larger number from the smaller.



Second step: Find the differences between the numbers in the corners

3. The next step is finding the differences between the earlier differences, making them into new corners, then finding the

[1] Zero is the lower bound of the whole numbers and fractions; there are no whole numbers or fractions less than zero, so zero is the inevitable ending point for race games and diffies.

[2] Liping Ma, *Knowing and Teaching Elementary Mathematics*, 1999 and 2010.

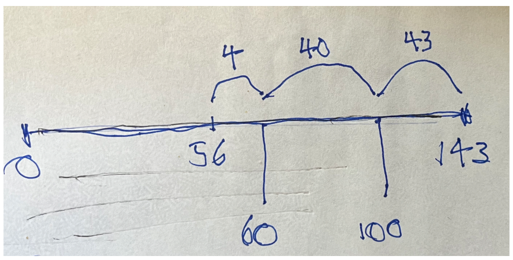
[3] Experiences of having done race games are helpful for students to keep in mind so that standard adding and subtracting, along with the trading and regrouping will be *just like* work they've already done with base ten blocks. Some young students even find it useful to use base ten blocks with their pencil on paper calculations.

[4] Some of the mathematics lurking beneath the routine doing of a diffy is the Trichotomy Law: Given two numbers A and B, one and only one of three things will be true: $A = B$, $A < B$, or $A > B$.

differences between those differences, and so on, working your way down, difference by difference, shell by shell, until all four of the differences are 0. (See the snapshot of a finished diffy above.)

Adding up

Here's an example of finding a difference by "adding up". For many of us, the default way of finding a difference is by subtracting. To find the difference between 143 and 56, we enter 143 on a calculator and take away 56. But if we don't have any electronic prostheses to help us, we have to do the subtraction on paper or in our head. But keeping in mind that subtraction depends on an addition relationship, we can easily see and find the gap between 56 and 143, as sketched on the number line below, that conveniently shows both the adding up from 56 to 143 and the taking away of 56 from 143, leaving 4+40+43=87.



We sketch of a number line and roughly locate 0, 56, and 143.

Working up from 56...

Adding 4 gets us to 60.
Adding 40 more gets us to 100.
And adding 43 more gets us to 143.

So the gap (aka the difference) must be 4 + 40 + 43 = 87.

We could also use (or imagine) a 1-100 grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

On a 1-100 grid, we locate 56

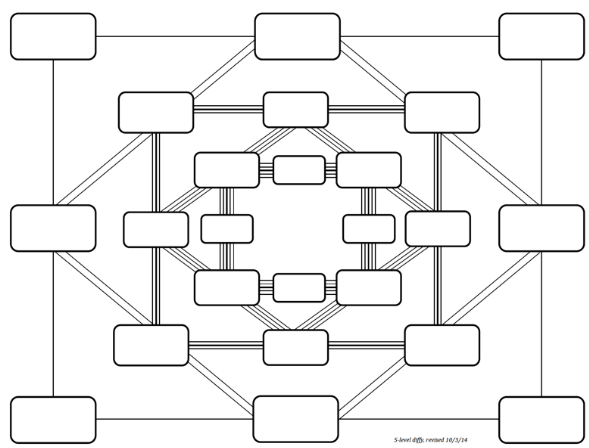
Working up from 56...

The next ten, 60, is 4 steps away.
After that, 100 is only 40 steps away.
Then 143 will be 43 steps away.

So the difference must be
4 + 40 + 43 = 87.

Diffy template & links

Some people try to do a diffy from scratch on a blank sheet of paper and get lost in a tangle of lines. So I made a blank diffy with five levels that students could use as a template. Some diffies have more than five levels, a few have less, but I've found that this template, maybe with an extra level at the end, will work for most diffies.



Download a blank diffy template from https://larrythemathguy.com/wp-content/uploads/2022/06/diffy_5-level_blank.pdf

You can do diffies with any whole numbers, fractions, or mixed numbers, large or small. For links to some downloadable PowerPoints that show the process step-by-step, visit larrythemathguy.com. Here are two:

3-digit whole number diffies (000-999) https://larrythemathguy.com/wp-content/uploads/2022/06/Diffy-3-digit.pptx
Mixed number fraction diffies https://larrythemathguy.com/wp-content/uploads/2022/06/diffy-mixednums-halves-3rds-6ths.pptx

Larry Francis, revised 9/7/2024