Race to 250 and back with 10-sided dice and base ten blocks

by Larry Francis

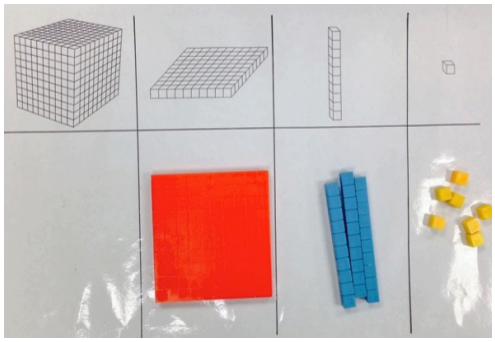
All students need to "get" base ten place value numeration and be able to use it fluently: 10 ones make a ten, 10 tens make a hundred, and so on. And all of us teachers need to know how to help them *get it*. For students to become something like native speakers of base ten, they need to be in lots of natural, uncontrived interactions and conversations. But how can we give our students enough practice—and they'll need a lot—to get base ten in their bones, as it were, including of course a fluent understanding of regrouping (aka trading[i])—while avoiding the tedium that so often accompanies lots of practice? For me, race games with ten-sided ones and tens dice are a big part of the answer: invite Lady Luck in. No one knows what will come up when the dice are rolled—any number from 00 to 99—so everybody has to pay attention. And since Lady Luck is driving, anyone can win, the strugglingest[ii] kid could beat the teacher, even the principal! Composition, decomposition, adding, and subtracting, need to become routine for students to "get" and "do" arithmetic. Playing race games, like race to 250 and back, is an enjoyable and effective way to do exactly that—with the 10-sided dice providing uncertainty and spice!

Tens die (00-90)
and ones die (0-9)

participants—two players and a banker

- **The players** need to be able to count to ten since they'll be "building" the numbers they roll by putting together the right amount of base ten blocks (tens and ones) and adding that amount to their total.
- **The banker** should have played the game at least a few times before and should be able to count to 10 by ones and to 100 by ones and tens. The banker reminds players to wait for the previous player's turn to end before starting their own turn, makes sure numbers are read and built correctly, makes sure no player ends a turn with more than 9 blocks of any denomination on their mat, keeps the game moving along, checks the numbers being built and the results of all the additions or subtractions, and resolves any disputes. The banker, often the teacher, should be someone who has played the game enough to manage the flow and keep it going, ideally with a kind of play-by-play commentary to keep students engaged with the arithmetic and probability involved. The banker checks the builds and trades for accuracy, but the banker doesn't make change: players have to submit their own trade requests to the banker. (And all trades must be fair—equal amounts for equal amounts.) One more responsibility for the banker: If a game gets interrupted, it's the banker's responsibility to write down each player's total, whether the game is going up or going down, and which player should get to take their turn when play resumes.

materials


Place value mat with 137 in base ten blocks

- 2 ten-sided dice: ones (0...9) and tens (00-90)
  The outcome of a roll of the dice give you the numbers in expanded form.
  The dice on the right show 70+6 =76
- 2 place value mats—one for each of the two players
  The place value mats are where the players "build" the number they rolled.
  The number in the graphic on the right would be built with 7 tens and 6 ones.
- base ten blocks
  - 30 "ones" (so you have a few extras)
    "Ones" are centimeter cubes, also known as "units"—the little yellow blocks in the snapshot to the right.
  - 30 tens (so you have a few extras)
    "Tens" are 1cm x 1cm x 10 cm blocks, also known as "longs"—the long blue blocks on the right.
  - 5 hundreds (so you have enough for when both players are over 200 and one player rolls 90 something, which puts them over 300)
    "Hundreds" are the 1cm x 10cm x 10cm blocks, also known as "flats"—the square orange blocks on the right.

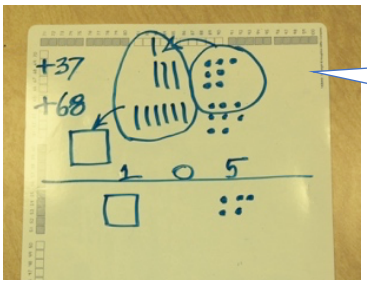
how to play

- A race game has two parts—the race going up and the race going down. In the up to 250, players (usually 2) take turns rolling the dice, building the number they just rolled with base ten blocks on a corner of their place value mat, then *adding* that number to the total they already have in the ones, tens, and hundreds columns of their mat, trading as necessary (so no player has more than 9 of any denomination on their mat) until somebody gets to 250 and wins. In the race going back down, players take turns rolling the dice and *taking away* the number rolled from their total until somebody gets to 0 or has an amount to take away that is bigger than the total on their place value mat.
- **Part 1: the race up to 250**, players take turns getting numbers and adding them to their running totals until somebody reaches or exceeds 250 and is the winner.
    - Both players start with 0, that is, they have no blocks on their place value mat.
    - The player who gets to take the first turn rolls the dice to get a number, and their turn begins.
    - The player builds the number they rolled with some base ten blocks. (Usually there is a "banker" who verifies that the number built is the number that was rolled.)
    - The player adds this new number to their previous total, "trading in" as necessary (since base ten place value notation requires that there be only a single digit in each place)
        - So whenever a player's total has over 9 of any denomination (i.e., 10 or more), they must trade in those 10 blocks with the banker for one of the next greater denomination
        - ten ones for one ten, ten tens for one hundred
    - When the player has finished any trading, so that they have no more than 9 blocks in any column of their place value mat, they read the number they have built, and if it's less than 250, it's the next player's turn.
    - Players take turns like this, rolling dice, building the number rolled, adding that number to their totals, saying the total out loud, checking to see if their number equals or exceeds 250, and then giving the dice to the next player to begin their turn, until a player reaches—or exceeds—250,
    - Then that player is the winner; the other player is the loser. No second chances; no extra rolls.

- **Part 2: the race back down to 0, starts at the moment when Part 1 ends:**
    - When one player reaches 250 or above in Part 1, Part 1 is over and Part 2 begins: the game changes to a race down to 0. Players keep their existing totals from the final results of the race going up, so the loser of the race going up to 250 gets to roll first on the way down to 0, which gives them a head start over the winner (A typical example might be 273 for the winner and 216 for the loser.)
    - Players take turns rolling the dice to get numbers, as before. But now, on the race down, instead of building numbers to *add* to their total players have to *take away* the number they roll from their previous total, until one player gets down to 0 or rolls a number greater than the total they have on their mat, since 0 is as low as you can go in the whole numbers.
    - The player has to build the number they rolled from their existing total of base ten blocks on their place value mat and then has to give that number to the banker. Sometimes that isn't easy: there are 3 possibilities:
        - (1) [Easy] The number a player rolls is less than the total of blocks on their place value mat and they have enough tens and ones to give the banker what they have rolled, so they go ahead and do it, and the game goes on.
        - (2) [Regrouping] The number a player rolls is less than the total they have on their place value mat, but they don't have enough tens or ones to build the number they rolled and give it to the banker. (The banker does not make change and only allows fair trades.) So, for example, if a player has 123 and they roll 65, they won't be able to give the banker 6 tens and 5 ones; they don't have 6 tens or 5 ones. They will need to trade. So they trade their hundred to the banker for 10 tens. Now they have 12 tens, enough to take away 6 tens, but they won't have enough ones until they trade in one of their tens for 10 ones. So they trade one of their tens to the banker for ten ones. Since they now have 11 tens and 13 ones they can give the number they rolled, 65 (6 tens and 5 ones) to the banker. That leaves them with 58 remaining on their mat, closer to 0 but still not there, so the game goes on.
        - (3) [We have a winner!] The number a player rolls is greater than the total amount they have on their mat: game over; they win. (Remember that we're in the whole numbers here, where you can't take away a greater number from a smaller number. Zero is the lowest you can go.)
    - As always, when a player rolls the dice, the player needs to announce the total rolled. And at the end of their turn, a player needs to announce their new total and say how it should be written.

Comments, observations, hints, tips, extensions

- Running out of time before the game is over is bound to happen on occasion, and it can make a good teachable moment, showing the purpose behind writing down numbers so they can be recalled later and built back just as they were—translating back and forth from the expanded form captured by the ones and tens dice to hands-on geometrical representations (base ten blocks) to numerical symbols and back again.
    - The banker needs to write down the players' current totals, or ask each of them to write down their scores and names on a single sheet of paper
    - with an arrow showing whether the race was going up or down
    - and with a star showing whose turn it will be when play resumes.
- Play-by-play chatter makes a game like a sporting event and prompts kids speculations about the mathematics
    - Who's in the lead? How far ahead?
    - How many more do you need to get to the next hundred? How many more to win?
    - Do you think you can get that many on your next roll? Is it even possible?
    - Are you so far back that you're doomed to lose? What do you think?
    - and so on, so it's like a horse race or a football match, but it's about math.
    - You might like taking a look at "The Play's the Thing: Mathematization as Dramatization" https://www.academia.edu/4185582/The_Play_s_the_Thing_Mathematization_as_Dramatization
- Race games show that counting is important and give students plenty of practice doing it:
    - players have to count the ones or tens to build (geometrically) what the ones and tens dice show (symbolically).
- Students learn to obey the conventions of base ten place value numeration: no more than 9 in any place.
- Translating from numerical symbols to base ten blocks to oral language helps students avoid these all-too-common errors:
    - Some young students who are learning to read and write, learning to track left-to-right, and learning to use sound-symbol correspondence, quite logically write "thirty-five" by first writing "30" and then "5"—305.
    - "Seventeen" often gets written quite logically too, spelled like it sounds, going left to right, starting naturally with a 7, followed by a 1 to denote "teen"—71.
    - Since at the end of each turn in a race game, players have to read their new total and say how they would write it, race games using a place value mat can help with that too.
- Students playing base ten race games get plenty of practice in making tens. A former colleague, a longtime middle school math teacher, told me once that all the struggling students he had tried to help over the course of his three-decade career in elementary and middle school math shared a common characteristic: they struggled with what he called "making tens". Obviously, since our modern world swims in base ten numeration, such students are doomed to struggle with math forever. Race games involve lots of counting, especially counting denominations of base ten blocks to see if they add up to 10, playing race games can help kids get comfortable with the basics of base ten, with plenty of practice taking the place of preaching and immersive activity taking the place of rote instruction.
- The unit for Race to 250 is the unit cube: 1cm x 1cm x 1cm. The base ten blocks make it easy to show and see equalities of length, area, and volume. 10 unit cubes have the same length, area, and volume as 1 "ten" (aka 1 "long"). And 10 "tens" (longs) have the same length, area, and volume as 1 "hundred" (aka 1 "flat"). Other manipulatives like beans and bundles of bean sticks, play money, straws, etc. don't have the same compelling geometry as base ten blocks, and they don't translate as nicely into activities that feature the area model for multiplication and division, partial products, and the distributive property, all of which students will face later in their school's math curriculum.
- Consider teaching kids to make a schematic sketch (shown below) of what they're doing with the base ten blocks. Kids can follow and summarize work with the blocks without the clutter, and expense of having a set of blocks for each individual.



This player had 37 and then rolled 68 (6 tens and 8 ones).
When added to the player's total, they had 9 tens and 15 ones.
- 10 ones were traded for 1 ten.
- 10 tens were traded for 1 hundred.

The result is shown schematically as 1 flat, no longs, and 5 units, and symbolically as 105.

Rules

Wait your turn.
Trades must be equal (fair).
Shake up the dice before you roll; don't just lay them down.
Do one thing at a time.
For example, if you have 87 and roll 25, you might get the bright idea of just grabbing your 8 tens and 5 ones and give them to the banker, snatch a hundred from the bank, and then take 12 from the bank. While it is true that 87 + 25 = 87+ (13 + 12) = 87 + 13 + 12 = (87+13) + 12 = 100 + 12 = 112, as a teacher I forbid it since it involves too many things happening too fast with too much left unsaid. I've seen sophisticated students do—or at least propose—things like this, leaving slower students back in the dust, totally mystified, making arithmetic seem to them like hocus-pocus as the more sophisticated student elided a bunch of steps in the regrouping. So I outlawed the clever stuff and settled on the rule ***one thing at a time***:

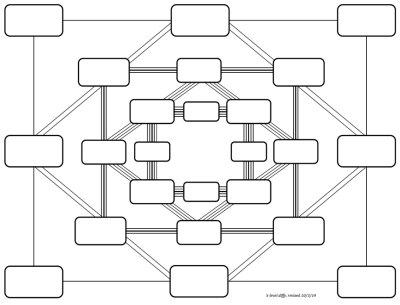
- roll 25,
- build it (2 tens and 5 ones)
- add it to your previous total (which was 87)
- now you have 10 tens and 12 ones.
- trade 10 tens for one hundred and 10 ones for one ten, giving you a new total of 112 (one hundred, one ten, two ones)

The banker must make *exact change*.

Some students (rather grandly) want to trade, for example, 12 ones for a ten.
The banker must not allow this, of course, since 12 ≠ 10

Miscellaneous endnotes, other games, and extensions

Race games can be played with any numbers to the right of 0 on the number line. You go up to a target number until somebody reaches it, then you race back down to zero. So whole numbers, positive fractions, and mixed numbers. Here are some modifications. “Little" numbers: race to 20 with 6-sided die, race to 50 with 10- or 12-sided die, race to 100 with a 20=sided die. “Big" numbers: race to 2500 with ones, tens, and hundreds dice.



a blank diffy

race games with fraction families

sixths family (0, 1/6/, 1/3, 1/2, 2/3, 5/6, 1) with a sixths die and pattern blocks or DIY tagboard pieces
eighths family (0, 1/8/, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1) with DIY tagboard pieces and an eighths die
tenths family with base ten blocks and tenths die
twelfths family with a blank 12-sided die (write in the fractions yourself) and DIY tagboard pieces
decimal fractions with base ten blocks and decimal fractions dice
(Keep in mind the limits the Common Core State Math Standards puts on fractions in grades 3 and 4.)

Sources I use for base ten blocks and dice are mainly *hand2mind* and *amazon*
Diffies are a great extension to race games. (I hope to write up something on diffies soon.)

How did I come up with 250 for the basic game—rather than some other number? We use a tens die and a ones die, so our range of possible rolls is 0…99. Figure an average roll of 50. I didn't want the game to go too fast *(what was that about?)* or too slow *(yawn!).* 250 seemed like a number that would give students a little more exposure to trading in ten tens for a hundred than racing to 100 or 200. I figure 5 turns from each player each way would be about right. 5x50=250. I use the same formula for the other race games, racing up to 3 in the fractions and up to 2500 in the thousands.

I first heard the phrase "race games" and learned how to play them probably in 1986 when I started teaching K-2 for Dr. Lee Jenkins at Maple Elementary School in Hesperia, California. I might have heard the phrase and learned a little about race games in 1982 or 1983 when I heard Lee's presentations at Oregon State University's Education Department's "Playful Learning" conferences.

I owe a great debt to Hung-Hsi Wu, Professor of Mathematics Emeritus at U.C. Berkeley. His three-week Mathematics Professional Development Institutes (2007-13) and his *Understanding Numbers in Elementary School Mathematics*, plus innumerable conversations over the years, have helped me see how the race games I've been playing and refining for so long fit into the big picture of K-12 mathematics instruction.

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[i] akin to Liping Ma's composition and decomposition (*Knowing and Teaching Elementary Mathematics)* composition, also to borrowing and carrying.
[ii] One of the terms I probably picked up from my ESL students back in the day.