The Story of 7's

Start with zero. You don't have any 7's yet. Obviously, that's zero copies of 7: You got nuthin'.
So 0 x 7 = 0

Get a 7 to add to your 0. Now you have a 7, but zero needs 10 to make 10 (duh), so you don't turn that corner: 0 + 7 = 7.
So 1 x 7 = 7

Get another 7 to add to your 7. Seven needs 3 to make 10, so you will turn the corner:
Think of the new 7 as (3 + 4); then you have 7 + 3 + 4 = (7 + 3) + 4 = 10 + 4 = 14.
So 2 x 7 = 14

Get another 7 to add to your 14. Now you have 3 copies of 7. Think of 14 as (10 + 4). 4 needs 6 more to make 10, and 7 is more than 6, so you know you'll turn that corner.
Think of the new 7 as (6 + 1), then you have 10 + 4 + (6 + 1) = 10 + (4 + 6) + 1 = 10 + 10 + 1 = 21.
So 3 x 7 = 21

Get another 7 to add to your 21. 1 + 7 = 8, so you know you won't make it to 30.
Now you have 20 + 1 + 7 = 20 + 8 = 28.
So 4 x 7 = 28

Get another 7 to add to your 28. Eight needs just 2 more to make 10, so you know you'll get to 30—and beyond it. Think of 28 as 20 + 8 and your new 7 as (2+5).
Then you have 20 + 8 + (2 + 5) = 20 + (8 + 2) + 5 = 20 + 10 + 5 = 30 + 5 = 35.
So 5 x 7 = 35

Get another 7 to add to your 35. Five needs just 5 more to make 10, so you know you'll get to 40—and beyond it—this time. Think of 35 as 30 + 5 and your 7 as (5 + 2)
Then you have 30 + 5 + (5 + 2) = 30 + (5 + 5) + 2 = 42.
So 6 x 7 = 42

Get another 7 to add to your 42. That's 40 + 2, as you know. Two needs 8 more to make 10, and you've only got 7, so you won't make it to 50. 40 + 2 + 7 = 40 + (2+7) = 40 + 9 = 49
So 7 x 7 = 49

Get another 7 to add to your 49. That's 40 + 9, as you know. Nine just needs 1 more to make 10, and you've got 7, so you know you'll go beyond 50 this time. Think of your 7 as (1 + 6), so you have 40 + 9 + (1 + 6) = 40 + (9 + 1) + 6 = 56.
So 8 x 7 = 56

Get another 7 to add to your 56. 56 = Again, you think of 56 in expanded form, as 50 + 6. Six needs 4 more to make 10, so you know you'll make it past 60. Thinking of 7 as (4 + 3), you have
50 + 6 + (4 + 3) = 50 + (6 + 4) + 3 = 63
So 9 x 7 = 63

Get another 7 to add to your 60 + 3. Three needs 7 more to make 10—and 7 is exactly what you have more of! So you have 60 + 3 + 7 = 60 + (3 + 7) = 60 + 10 = 70.
So 10 x 7 = 70